$$2(x-r)+2(mx-r)+2r=x+mx+x\sqrt{(1+m^2)}.$$

$\therefore\ r=\frac{1}{2}x[1+m-\sqrt{(1+m^2)}]$.

$GH=\frac{1}{3}BC=\frac{1}{3}mx,\ KH=\frac{1}{2}AB-HB=\frac{1}{2}x-r$.

$\therefore\ KH=\frac{1}{2}x[\sqrt{(1+m^2)}-m)],\ GK=\frac{1}{3}CK=\frac{1}{3}x\sqrt{(\frac{1}{4}+m^2)}$.

$\therefore\ KG^2=GH^2+KH^2$, or $\frac{1}{9}x^2(\frac{1}{4}+m^2)=\frac{1}{9}m^2x^2+\frac{1}{4}x^2[\sqrt{(1+m^2)}-m]^2$.

$\therefore\ \frac{1}{4}[\sqrt{(1+m^2)}-m]^2=\frac{1}{36}$, or $\sqrt{(1+m^2)}-m=\frac{1}{3}$. $\therefore\ m=\frac{4}{3}$.

$\therefore\ AB : BC=3 : 4$, and $HB : AB : BC : AC=1 : 3 : 4 : 5$.

Also solved by *J. C. NAGLE.*

DIOPHANTINE ANALYSIS.

60. Proposed by G. B. M. ZERR, A. M., Ph. D., President and Professor of Mathematics, The Russell College, Lebanon, Va.

It is required to find six positive numbers, such that if each be diminished by five-half times the fifth power of their sum the six remainders will be rational fifth powers.

Solution by the PROPOSER.

Let u, v, w, x, y, z, be the six numbers required, and let $u+v+w+x+y+z=s$.

Then $u-\frac{5}{2}s^5=h^5s^5/q^5,\quad v-\frac{5}{2}s^5=k^5s^5/q^5,\quad w-\frac{5}{2}s^5=l^5s^5/q^5,$

$x-\frac{5}{2}s^5=m^5s^5/q^5,\quad y-\frac{5}{2}s^5=n^5s^5/q^5,\quad z-\frac{5}{2}s^5=p^5s^5/q^5,$

Adding these six equations we get

$$s-15s^5=(s^5/q^5)(h^5+k^5+l^5+m^5+n^5+p^5).$$

Let $h^5+k^5+l^5+m^5+n^5+p^5=q^5.\quad \therefore\ s=\frac{1}{2}$.

$\therefore\ u=\frac{1}{32}[\frac{5}{2}+(h^5/q^5)],\quad v=\frac{1}{32}[\frac{5}{2}+(k^5/q^5)],\quad w=\frac{1}{32}[\frac{5}{2}+(l^5/q^5)],$

$x=\frac{1}{32}[\frac{5}{2}+(m^5/q^5)],\quad y=\frac{1}{32}[\frac{5}{2}+(n^5/q^5)],\quad z=\frac{1}{32}[\frac{5}{2}+(p^5/q^5)].$

Let $h=4$, $k=5$, $l=6$, $m=7$, $n=9$, $p=11$, $q=12$.

$u=\frac{1217}{15552},\ v=\frac{625205}{7962624},\ w=\frac{81}{1024},\ x=\frac{638887}{7962627},\ y=\frac{2803}{32768},\ z=\frac{783131}{7962624}.$

Let $h=5$, $k=10$, $l=11$, $m=16$, $n=19$, $p=29$, $q=30$.

$u=\frac{19441}{248832},\ v=\frac{1217}{15552},\ w=\frac{609110051}{7776000000},\ x=\frac{38624111}{486000000},\ y=\frac{632226099}{7776000000},\ z=\frac{812611149}{7776000000}.$

AVERAGE AND PROBABILITY.

58. Proposed by HENRY HEATON, M. Sc., Atlantic, Iowa.

From a point on the surface of a circle two lines are drawn to the circumference. Required the average area that may be cut from the circle in this way if the lines are supposed to be drawn at equal angular intervals.

Query I. How does this differ from problem 32?

Query II. Is *sector* the proper word to use for the surface thus cut off?

Query III. Is it absolutely correct to use the word *random* in average problems?

I. Solution by the PROPOSER.

For each pair of lines a second pair may be drawn in opposite directions, dividing the surface of the circle into four portions each of which is included between two of the lines and the circumference. Hence the whole number of surfaces thus cut off may be arranged in sets of four such that the areas of each set shall equal the area of the circle. Hence the average required is $\frac{1}{4}a^2\pi$, where a is the radius of the circle.

Query I. As problem 32 does not describe how the lines are to be drawn to form the "sector" this is a particular case of that problem.

Query II. This query was proposed for information. Some one may be able to give authority for the use of the word in this sense. It is contrary to the usual definition.

Query III. It is the opinion of the writer that the use of the word *random* in average problems is the result of confusion of ideas, and although sometimes convenient is never correct.

II. Solution by G. B. M. ZERR, A. M., Ph. D., President and Professor of Mathematics, The Russell College, Lebanon, Va.

Let P be the given point. Through P draw the two chords MN, SR dividing the surface of the circle into the four surfaces A, B, C, D.

Then $A+B+C+D=\pi r^2$.

Since P can be taken anywhere on the surface of the circle and the lines MN, SR can make any angle from 0 to π, the average area of A=average area of B=average area of C=average area of D.

$\therefore\ A=B=C=D=\frac{1}{4}\pi r^2$.

After carefully examining problem 32 I am inclined to think the above result the true answer to that problem also.

59. Proposed by J. SCHEFFER, A. M., Hagerstown, Md.

A circle is rolling along a horizontal straight line. The uniform velocity of the center is v. Find the average velocity of a point of the circumference.

Solution by JOHN M. COLAW, A. M., Monterey, Va.; JOSIAH H. DRUMMOND, LL. D., Portland, Me.; M. E. GRABER, Mt. Vernon, O.; and the PROPOSER.

For the cycloid traced by the point, we have

$$\left.\begin{array}{l} x=a\theta-a\sin\theta \\ y=a-a\cos\theta \end{array}\right\},$$

$$dx=a(1-\cos\theta)d\theta\ ;\ dy=a\sin\theta d\theta.$$

$\therefore\ ds^2=dx^2+dy^2,=2a^2d\theta^2(1-\cos\theta),=2a^2d\theta^2(2\sin^2\tfrac{1}{2}\theta)$.

$\therefore\ ds=2a\sin\tfrac{1}{2}\theta d\theta$.

Now $OT=vt=a\theta$. $\therefore\ dt=(a/v)d\theta$.

$\therefore\ ds/dt=2a\sin\tfrac{1}{2}\theta d\theta\div(a/v)d\theta,=2v\sin\tfrac{1}{2}\theta$, the variable velocity of P.